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1990 J. Phys. A: Math. Gen. 23 4415

(http://iopscience.iop.org/0305-4470/23/19/023)

J. Phys. A: Math. Gen. 23 (1990) 4415-4416. Printed in the UK

COMMENT

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Received 13 June 1990

Abstract. The rational solution for the minimal representation of the quantum G_2 is obtained as a limit of the trigonometric one presented by Kuniba.

A rational solution to the Yang-Baxter equation can be obtained from a spectrum-dependent trigonometric one through an appropriate limit process [1-3]. The latter can be computed by a standard method of Jimbo [4]. The key in this method is to find out the quantum generators $X_\theta^{\pm}$ and H_θ associated with the maximal root θ. Their general expressions are not easily obtained, but for a given representation of a given quantum Lie universal enveloping algebra, it may be possible. Kuniba [5] found the expressions for the minimal (seven-dimensional) representation of the quantum G_2, and then obtained the spectrum-dependent solution $R(u)$ to the Yang-Baxter equation. It is straightforward to obtain the corresponding rational solution, which means that another solvable 175-vertex model may be constructed.

Multiplying a factor $\{q^{3u}(1-q^2)(1-q^8)(1-q^{10})\}$ for simplicity, we have

$$\begin{aligned}\check{R}_q(x) &= q^{3u}(1-q^2)(1-q^8)(1-q^{12})R(u)\\ &= (1-xq^2)(1-xq^8)(1-xq^{12})\mathscr{P}_{2\bar{\Lambda}_2}+(x-q^2)(1-xq^8)(1-xq^{12})\mathscr{P}_{\bar{\Lambda}_1}\\ &\quad +(1-xq^2)(x-q^8)(1-xq^{12})\mathscr{P}_{\bar{\Lambda}_2}+(x-q^2)(1-xq^8)(x-q^{12})\mathscr{P}_0 \qquad (1)\end{aligned}$$

where $x=q^{2u}$. We redefine the spectral parameter u as u/η where η is called the quantum parameter in the rational solutions. Taking the limit $q\to 1$, we obtain the corresponding rational solution $R(u,\eta)$ as follows:

$$\begin{aligned}R(u,\eta) &= P\check{R}(u,\eta)=\lim_{q\to 1} P\check{R}_q(q^{2u/\eta})/(1-q^{2u/\eta})^3\\ &= (1+\eta/u)(1+4\eta/u)(1+6\eta/u)P_{2\bar{\Lambda}_2}+(1-\eta/u)(1+4\eta/u)(1+6\eta/u)P_{\bar{\Lambda}_1}\\ &\quad +(1+\eta/u)(1-4\eta/u)(1+6\eta/u)P_{\bar{\Lambda}_2}+(1-\eta/u)(1+4\eta/u)(1-6\eta/u)P_0\\ &= \mathbb{1}+(3t+9\mathbb{1})\eta/u+(18\mathbb{1}+4P+18t+28P_0)\eta^2/u^2+24P\eta^3/u^3 \qquad (2)\end{aligned}$$

where P denotes the transposition,

$$P_\Lambda=(\mathscr{P}_\Lambda)_{q=1} \qquad \Lambda=2\bar{\Lambda}_2,\bar{\Lambda}_1,\bar{\Lambda}_2,0$$

$$(P_0)_{\mu\nu,\mu'\nu'}=\tfrac{1}{7}(-1)^{\mu+\mu'}\delta_{-\mu\nu}\delta_{-\mu'\nu'} \qquad \mu,\nu,\mu' \text{ and } \nu'=-3,-2,\ldots,2,3$$

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0305-4470/90/194415+02$03.50

and

$$t = \sum_a I_a \otimes I_a$$

where I_a are the orthogonal bases of G_2 with the convention that the square length of the longer root is 2 (see [5]). Obviously, we have

$$R(u, \eta)R(-u, \eta) = (1 - \eta^2/u^2)(1 - 16\eta^2/u^2)(1 - 36\eta^2/u^2)\mathbb{1} \tag{3}$$

Acknowledgment

The author would like to thank Professor Atsuo Kuniba for sending his offprint [5]. This work was supported by the Natural Science Foundation of China through the Nankai Institute of Mathematics.

References

[1] Jimbo M 1985 *Lett. Math. Phys.* **10** 63
[2] Hou Bo-Yu, Hou Bo-Yuan, Ma Zhong-Qi and Yin Yu-Dong 1990 Rational solution to Yang–Baxter equation in the octet representation *Preprint* BIHEP-TH-90-12
[3] Zhong-Qi Ma 1990 *J. Phys. A: Math. Gen.* **23** in press
[4] Jimbo M 1986 *Commun. Math. Phys.* **102** 537
[5] Kuniba A 1990 *J. Phys. A: Math. Gen.* **23** 1349